A model to estimate COVID 19 spread in Italy

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March 26, 2020

Population

Following Chenlin et al. (2020) , it is possible to divide the infected population into three sub-groups:

- $I_c : \mathbb{R}^+ \to \mathbb{Z}$, the **cumulative confirmed cases**
- $I_a : \mathbb{R}^+ \to \mathbb{Z}$, the **active undetected cases**
- $I_q : \mathbb{R}^+ \to \mathbb{Z}$, the **unconfirmed cases under quarantine**

The total cumulative cases is:

$$I(t) = I_c(t) + I_a(t) + I_q(t) \tag{1}$$

Model parameters in Italy

The table below shows the values of our model parameters at the beginning of the outbreak in Italy.

Variable	Value (Interval)	Definition
T	[0,15]	Incubation period from infection to symptoms
θ	[0,1]	Rate of confirmation once detected
β	0.29	Rate of infection
α	[0,1]	Probability for close contacts to be under quarantine

Net Reproduction rate

$[R_0]$ is the net reproduction rate, which is closely linked to the infection rate and it is used to measure the epidemic risk. Depending on its value, we can distinguish three epidemic phases:

- *Super-critical case:* $R_0 > 1$, epidemic has an exponential increase
- *Critical-case:* $R_0 = 1$, epidemic persists over time
- *Sub-critical case:* $R_0 < 1$, , after a period, epidemic is under control.

If $\theta = 1$ and $\alpha = 0 \Rightarrow R_0 = \beta \mathbb{E}[T]$.
If the quarantine is applied and the rate of confirmation is not perfect $0 < \theta < 1$, asymptotically we have:

$$\tilde{R}_0 \simeq \frac{\beta(1-\alpha)}{\theta} \mathbb{E}[T] \tag{2}$$

Enforceable Actions

To control the spread of the epidemic, we need to get to $\tilde{R}_0 < 1$. The enforceable actions are:

- Reduce the infection rate β by banning activities in public spaces and person-to-person contact
- Make confirmation rate θ as close as possible to 1 improving the accuracy of the diagnostic tests (transforming I_a in I_c)
- Increase quarantine rate α by tracking the confirmed cases and all their possible contacts (transforming I_a in I_q).

Data

To analyse the spread of COVID-19 in Italy, we used the Civil Protection data for the period 21 February - 17 March, 2020. Data are organized as follows:

- New Active cases
- Cumulative cases to date
- Daily tests

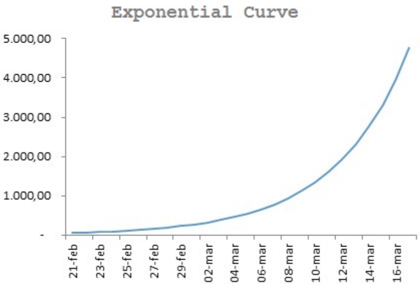
To study the Chinese trend, we used data provided by the European Centre for Disease Prevention and Control (EDCD), which report new daily cases in the period 19 January - 17 March 2020.

Model parameters for Italy

As virus spread follows an exponential curve, we built the theoretical infection curve using the sub-sample 21-29 February 2020 and regressing the logarithmic transformation of the historical series of confirmed cases in Italy (PosIta) over time, thus getting:

$$ln(PosIta) = 3.33595 + 0.25778t \quad (3)$$

which corresponds to an infection rate $\beta = 0.352$ and a reproductive number $R_0 = 2.46$.



Degree of quarantine measures (α)

α is determined on the basis of the increasing degree of the restrictive measures imposed by the Italian government to limit the spread of the infection.

Chenlin et al - Scale:

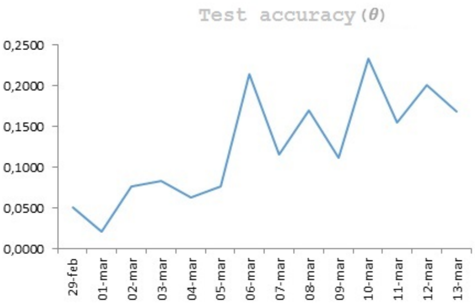
- Weak measures $= 0.2$
- Normal measurements $= 0.4$
- Strong measures $= 0.6$
- Very strong measures $= 0.8$

Date	Total cases	Restriction	Population (%)	α	Weighted (α)
23- Feb	221	Red zone creation	0.08	0.80	0.001
25- Feb	229	Urgent measures to contain infection	43.89	0.20	0.09
		Further measures to protect	0.10	0.80	
01- Mar	1,337	Risky areas	36.40	0.60	0.22
		Two levels of protection	4.72	0.75	
08- Mar	6,387	restrictions	95.30	0.60	0.70
09- Mar	7,985	Creation of a unique red zone	100	0.75	0.75
11- Mar	12,462	more Restrictions	100	0.80	0.80

Test accuracy (θ)

To estimate θ, we use a proxy obtained by dividing the numbers of confirmed cases by the total number of tests (TNT) carried out day by day:

$$\theta = \frac{PosIta_t}{TNT_t} \tag{4}$$



Simulation

Once the value to assign to each parameters was detected, we estimated the effect of each combination of the two measures in terms of infection reduction.
We considered the variation that occurs to the constrained theoretic curve with respect to the free one.

Date	Policy	α	Infection Effect θ	% variation w.r.t previous period
29- Feb	Parameters before any policy	0.0880	0.0769	-
01- Mar	Measures for risky areas	0.1713	0.2200	76.85%
08- Mar	Two levels of protected zone	0.1850	0.6115	15.55%
09- Mar	Creation of a unique red zone	0.1841	0.75	16.23%
11- Mar	More Restrictions	0.1891	0.800	20.00%

Comments

We also checked the effect of each policy, if applied from the very beginning. The results are as follows:

Policy	α	θ	Infection gain (%)
All measures	0.640	0.1716	91.1%
Only 1st march policy	0.220	0.0769	14.3%
Only 8th march policy	0.610	0.0769	57.2%
Only 9th march policy	0.750	0.0769	72.6%
Only 11th march policy	0.800	0.0769	78.1%
High accuracy tests	0.088	0.1891	59.3%

Once we assessed the containment effect of measures, we used three models to estimate the possible evolution of COVID-19 spread in Italy:

1 **If Italy is like China**

$$ContIta_{t+h} = -5.293 + 0.633 PosChina_{t-32+h} \tag{5}$$

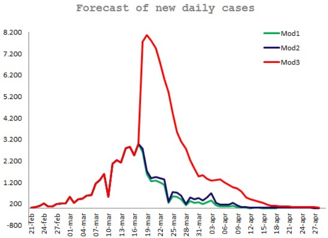
2 **Italian Restriction Model with full Compliance**

$$PosIta_{t+1} = \tilde{R}_0 \frac{\sum_{i=t}^{t-6} ContIta_i}{7} \tag{6}$$

$\tilde{R}_0$ is known, since its parameters are exogenously determined (α and θ by the measures, β arises from the trend of the infected in China).

3 **Italian Restriction Model with low Compliance** The same of model 2 but for the use of the theoretical β in order to define an upper limit in the absence of compliance.

Results



- The model proved to be a performing forecasting tool in terms of peak date estimation. Data are currently following the path of the frontier designed by the model.
- The degree of accuracy of forecasts is heavily dependent on several factors, for this reason the curve resulting from the model is then a frontier, which implies a min and max overlap with real data, according to the variations in the above mentioned parameters, in particular the level of compliance.